

CSE 2600

Intro. To Digital Logic & Computer Design

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Content Review

- [Site](#)
- Graded parts
 - Prep 1: “Due” yesterday (but still can be submitted)
 - Prep 2A: Due on next Mon, Aug 31
 - Hw1: Due on next Wed, Sep 2

Review & Expansion

Chapter 1 Sections

3. Digital Abstraction

4. Number Systems

5. Logic Gates

Abstraction

- Digital discipline
 - Discrete values: 0 and 1
 - Smallest unit of information: a binary digit. Also-know-as a Bit
- (Mostly) Starting at gate level

Counting

Decimal	Binary
00	000
01	001
02	010
03	011
04	100
05	101
06	110
07	111

Binary Basics: Number Line



Conversions: Place Value

Place Value: Base 2

To Decimal

Example: 110_2 (or 3'b110)

Digits	1	1	0
Place Value (Decimal)	4	2	1
Place Value In	2^2		
Expansion	1×2^2	$+1 \times 2^1$	$+0 \times 2^0$

Digit in decimal

Value of place in decimal

Sum them all

Hexadecimal

- Convenient, compact way to deal with binary
- Each hex digit = exactly 4 binary digits
- Sixteen digits:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Counting

Decimal	Binary	Hex
00	0000	
01	0001	
02	0010	
03	0011	
04	0100	
05	0101	
06	0110	
07	0111	
08	1000	
09	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

Counting

Decimal	Binary	Hex
00	0000	0
01	0001	1
02	0010	2
03	0011	3
04	0100	4
05	0101	5
06	0110	6
07	0111	7
08	1000	8
09	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hex Notations

- Programming: 0x followed by digits, like 0xAB7
- Verilog: *n'hdigits*

12'hAB7

Decimal number of BITS

h for Hex

Hex digits

Hex to Binary

- What is 12'hAB7 in binary?

Hex to Decimal

- What is 12'hAB7 in decimal?
 - Option 1: Convert to binary and then the binary to decimal
 - Option 2: Use place-value approach

Hex to Decimal

- What is 12'hAB7 in decimal?

What is 123 in hex?

- Greedy approach
 - A lot like binary, but have to ask “how many times does this go in...”

Place Value (Decimal)	256	16	1
Place Value In terms of Base	16^2	16^1	16^0

Why hex?

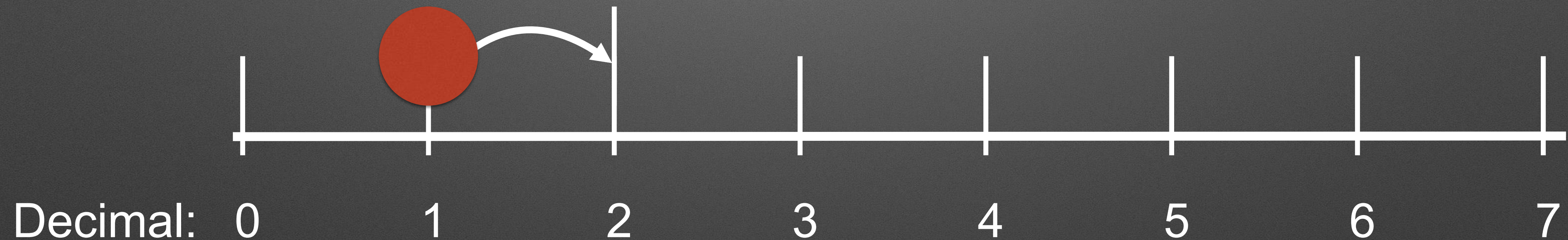
- Consider 32-bit numbers
 - 0xA764 1CD8
 - Vs. 1010 0111 0110 0100 0001 1100 1101 1000

Review & Expansion: Arithmetic

Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

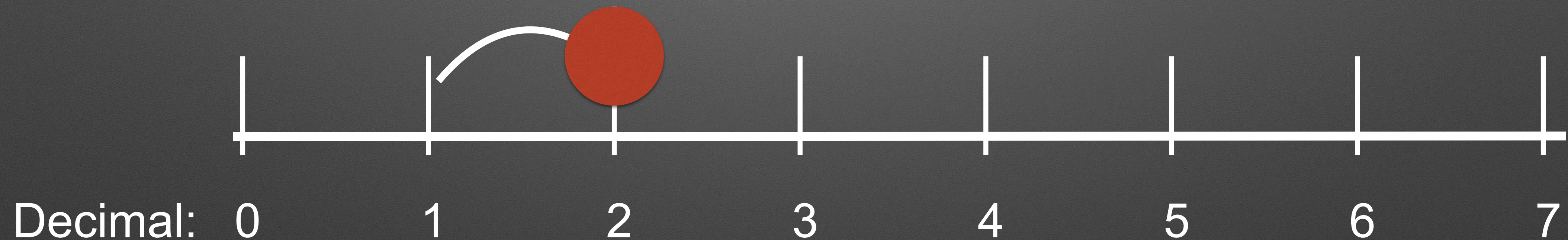
Ex: $1+2$



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

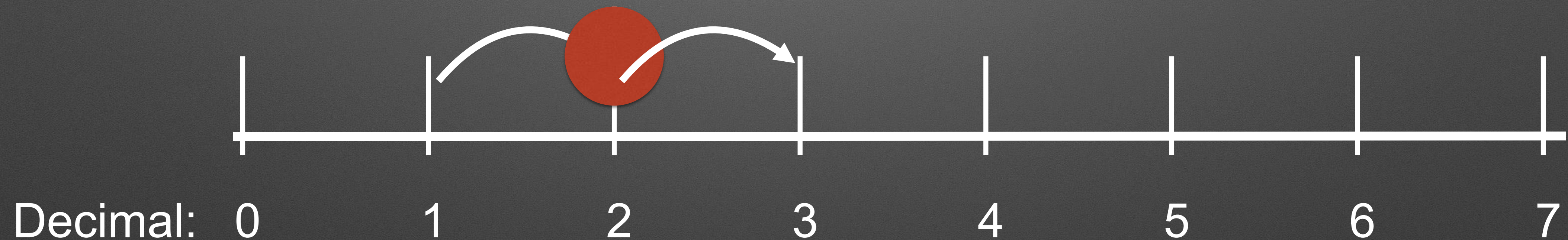
Ex: $1+2$



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

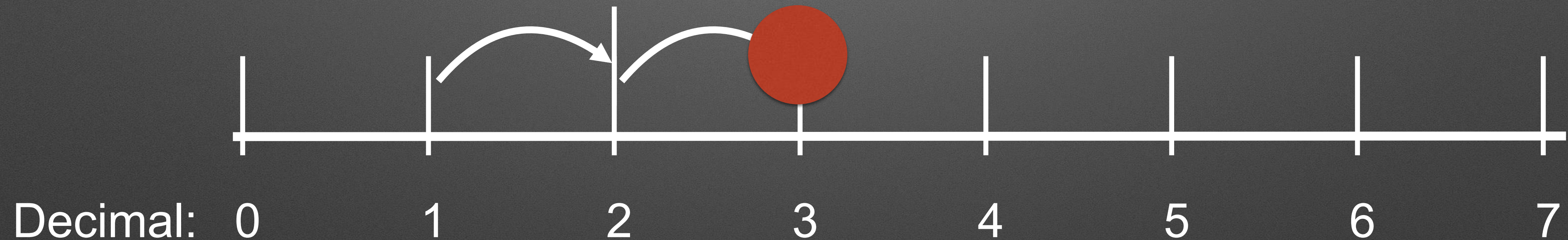
Ex: $1+2$



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

Ex: $1+2$



Binary Addition Rules

- Condensed
 - No ones: $0+0+0 = 00$
 - One one: $0+0+1 = 01$
 - Two Ones: $0+1+1 = 10$
 - Three Ones: $1+1+1 = 11$

Binary Addition Rules: Fully Elaborated

0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

Review Problem

- Add $4'b0110 + 4'b0010$

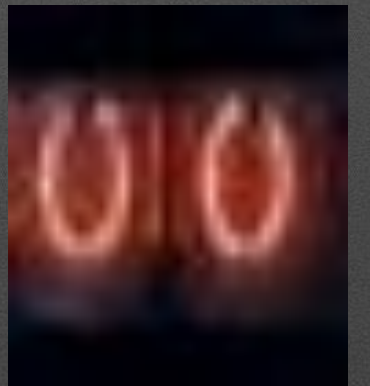
Fixed Width Arithmetic



Fixed Width Arithmetic



Fixed Width Arithmetic



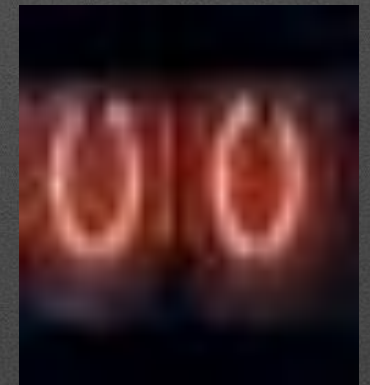
Fixed Width Arithmetic

- Consider the following problems:

- $1 + 2 =$

- $95 + 6 =$

- $80 + 20 =$



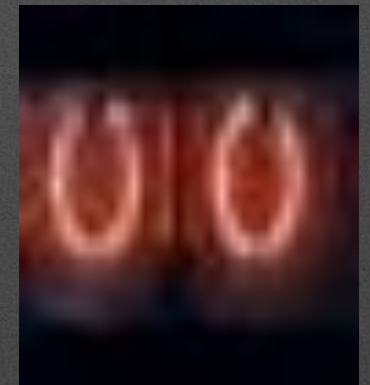
Fixed Width Arithmetic

- Consider the following problems:

- $01 + 02 = 03 \Rightarrow 03$
- $95 + 06 = 101 \Rightarrow 01$
- $80 + 20 = 100 \Rightarrow 00$

- Consider the following problems:

- $03 \text{ op number} = 03$
- $101 \text{ op number} = 01$
- $100 \text{ op number} = 00$



Fixed Width Arithmetic

- Consider the following problems:

- $01 + 02 = 03 \Rightarrow 03$

- $95 + 06 = 101 \Rightarrow 01$

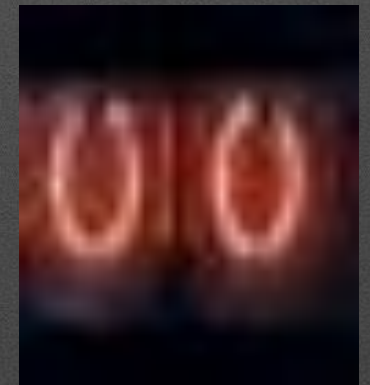
- $80 + 20 = 100 \Rightarrow 00$

- Consider the following problems:

- $03 \% 100 = 03$

- $101 \% 100 = 01$

- $100 \% 100 = 00$



Fixed Width Arithmetic

- $A + B$ With n -digit inputs and result in base b
- Result is $(A + B) \% b^n$

Modular Arithmetic & The Number Line

(Binary, 3-bit)



What's $1+2$?

Modular Arithmetic & The Number Line

(Binary, 3-bit)



What's $6+2$?

Full, binary number line (2^n values): Exceeding End Wraps!



What's 6+2?

Going Negative

- What about negative numbers?
 - Easy option: Encode the concept of a sign
 - 0 = positive
 - 1 = negative
 - Pick a bit to represent the sign

Example: 4-bit sign / magnitude

- Format

Place:	Sign	2^2	2^1	2^0
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- Sign first (common choice)
- What is the decimal value of the 4-bit, sign-magnitude
 - 4'b1001
 - 4'b0110

Really Negative

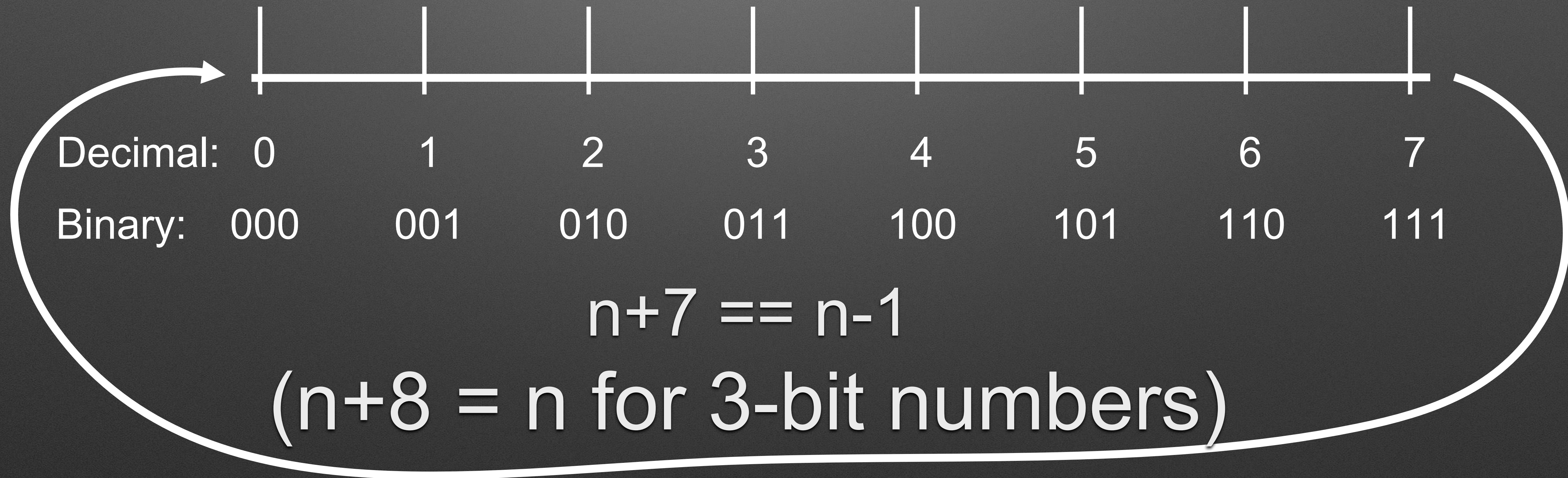
- What is the decimal value of the 4-bit, sign-magnitude
 - 4'b1000
 - 4'b0000
 - Weird
- What is $4'b1001 + 4'b0110$?
 - Arithmetic is messy with sign/magnitude!

UNSIGNED Again...

Challenge: Describe the result of $n+7$?



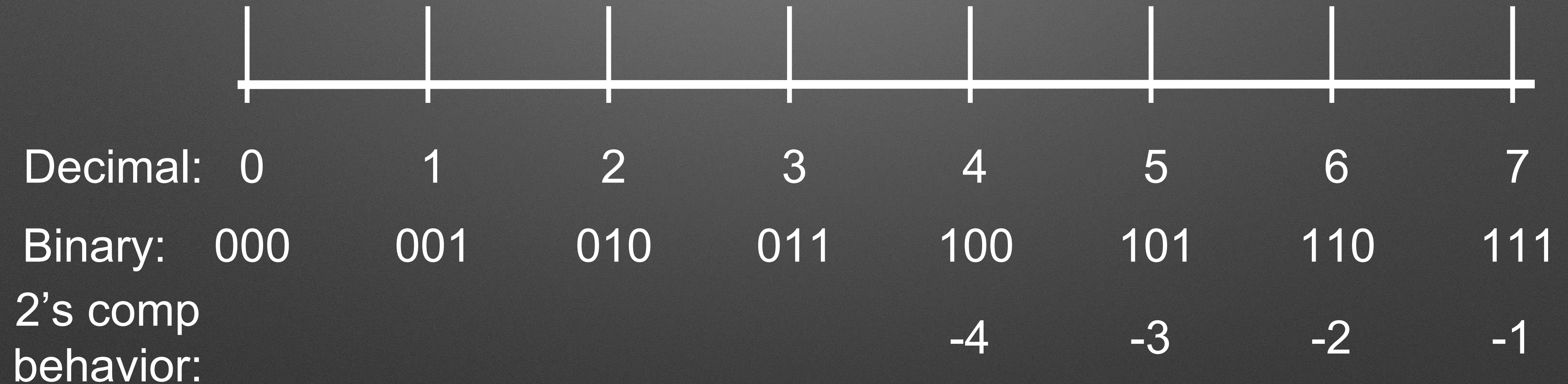
Challenge: Describe the result of $n+7$?



Challenge: How can you emulate $n-2$?



The Magic of Fixed Width numbers (modular arithmetic): Addition can emulate subtraction!



Two's Complement Sign Representation

The “Two’s Complement”

Dictionary

Definitions from [Oxford Languages](#) · [Learn more](#)

com·ple·ment



noun

/ˈkæmpləmənt/

1. a thing that completes or brings to perfection.
"the libretto proved a perfect **complement** to the music"

Similar:

accompaniment

companion

addition

supplement

accessory



2. a number or quantity of something, especially that required to make a group complete.
"at the moment we have a **full complement** of staff"

Similar:

amount

total

aggregate

contingent

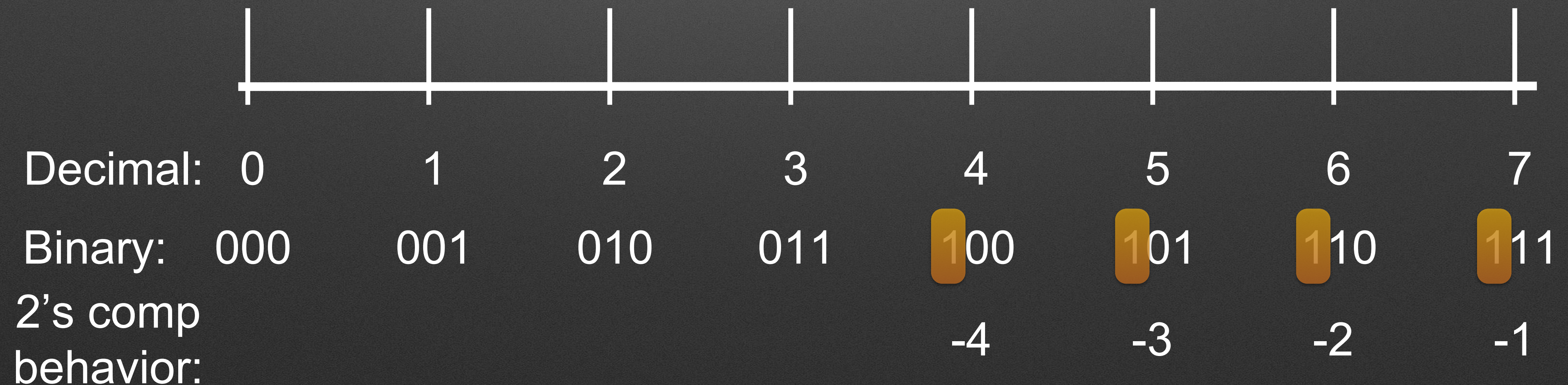
company

capacity

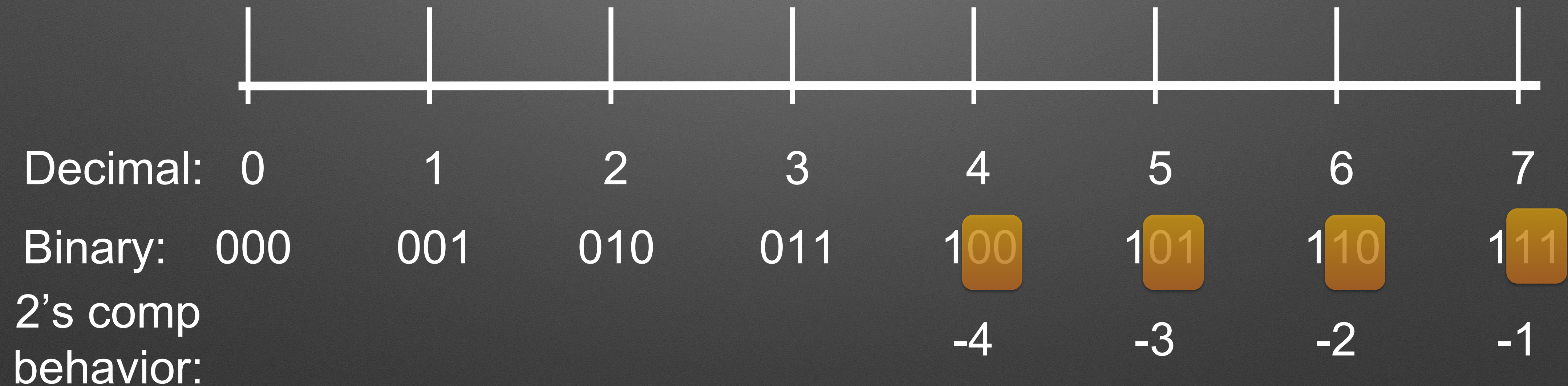


Dividing the Line

- It'll be nice if we have about half as many negatives as positives. We'll split the line in half, like we've already done.
- How can we identify if a binary number is positive or negative?
 - Ex: 010? Or 110?



What about conversions & place/value stuff?



Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the decimal value of the 3-bit, 2's complement numbers:

110

011

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the 3-bit, 2's complement representation of:

2

-4

-5

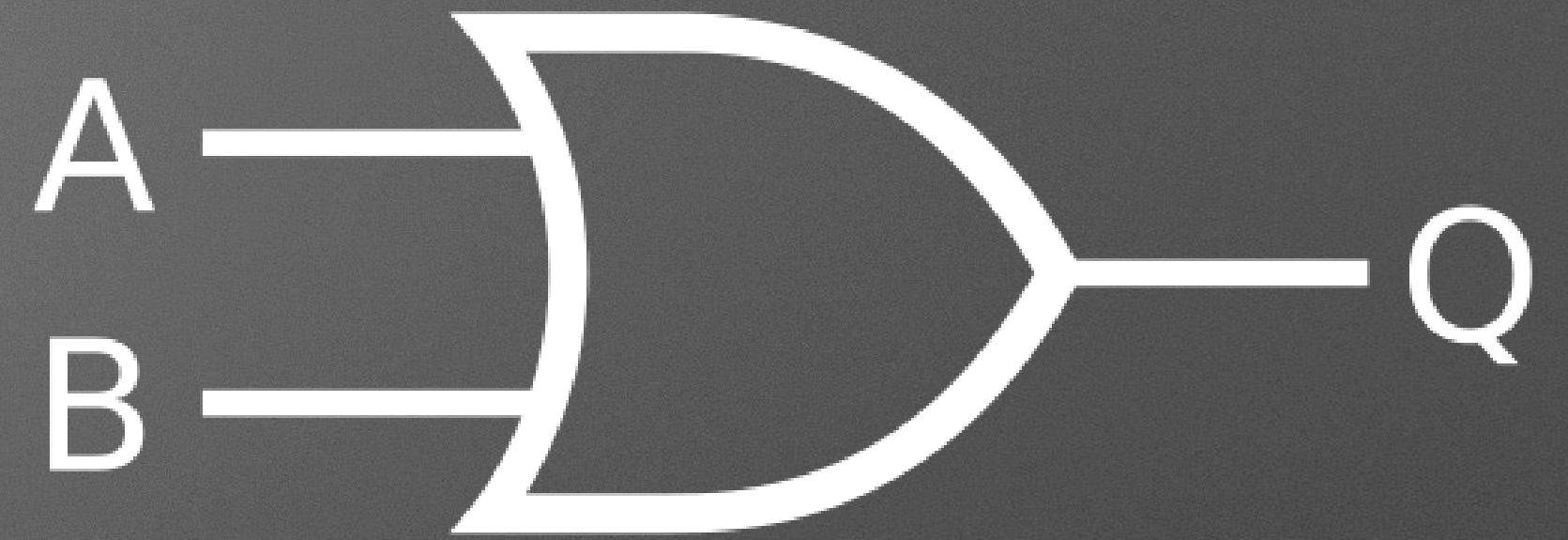
4-bit Two's Complement

- What is the decimal value of the 4-bit, Two's Complement Number
 - 0xA
 - 0x7

Gates & Tables

- We can describe the *final behavior* simple machines, like gates, with tables
 - Truth table
 - Row for each possible combination of inputs
(Binary; Fixed width counting)
 - Column for each input and each output

OR Gate



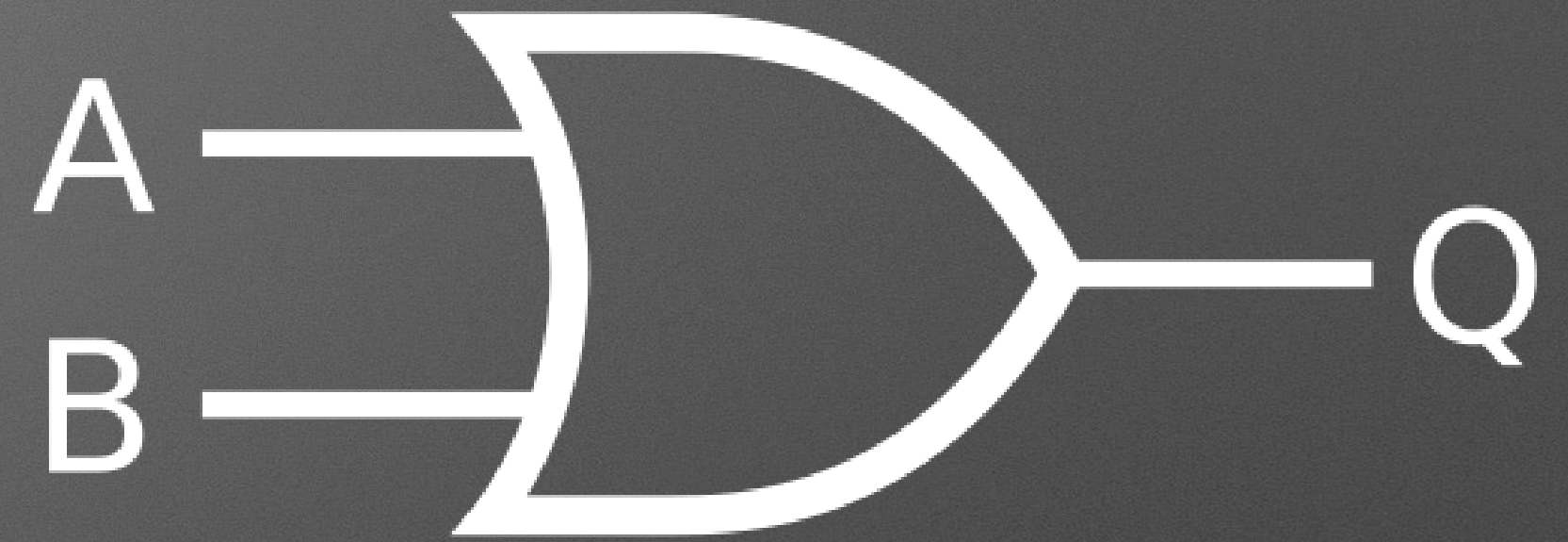
- How many rows in truth table?
- How many columns?

OR: Gate...A real machine

- Spintronics game: <https://upperstory.com/en/spintronics/>

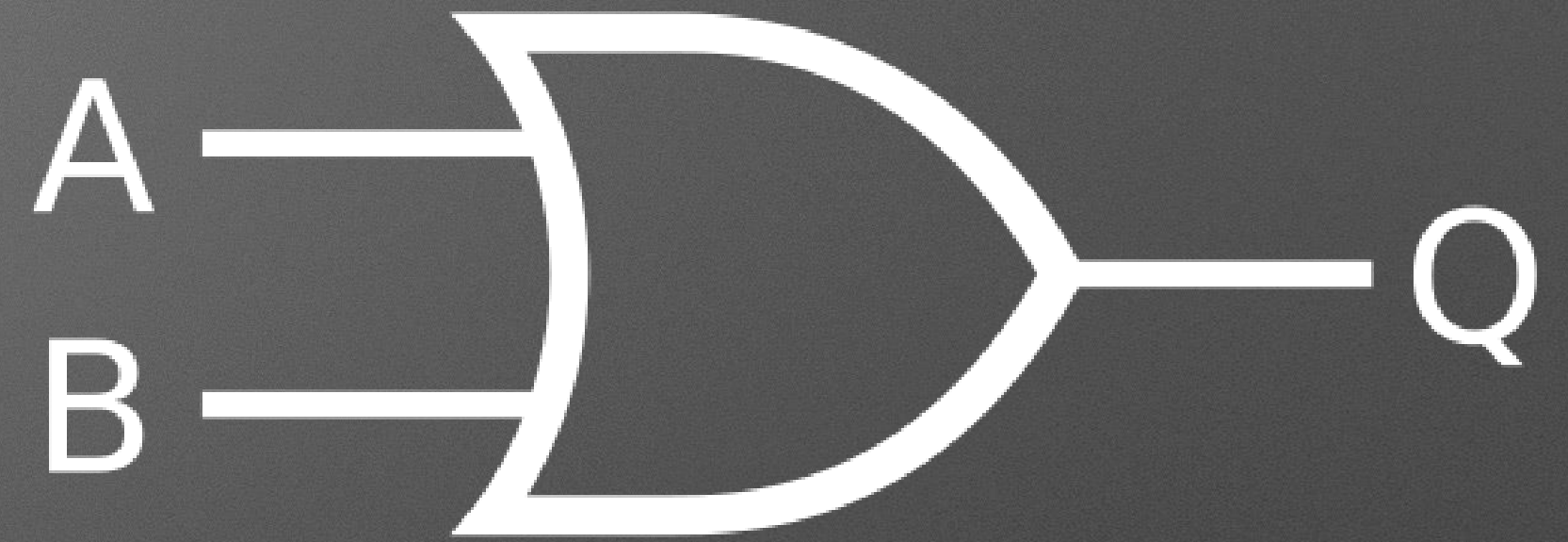


New Idea: Behavior OVER TIME



- Gate represents a machine of some sort machine
 - in the real world
- They are not instantaneous

The timing diagram



Intro to JLS

Coming Due Dates

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- Prep 2A: Due on next Mon, Aug 31
- Hw1: Due on next Wed, Sep 2

Questions

- “It’s hard for me to conceptualize how an entire computer runs on bits and how electrons make all of it happen.”
- “At what point does hiding details using abstraction become an issue?”
- “How did we discover the two’s complement? Is there a mathematical reason why it just works?”
- “How do the logic gates build up to a full computer?”
- “How does hardware actually distinguish between data and instructions in memory?”

