

CSE 2600

Intro. To Digital Logic & Computer Design

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This week

- In-person demos required for some credit on nearly all remaining assignments
- Hw#5A posted tonight

RISC-V Edition of book!

Studio 4B Review & Ch 5

Studio 4B

- Full-Adder: Behavioral variations in simulation
 - iCE40 Mapping
- Full-Adder: Column of number

Multi-bit and Ripple Carry

Wikipedia Animation

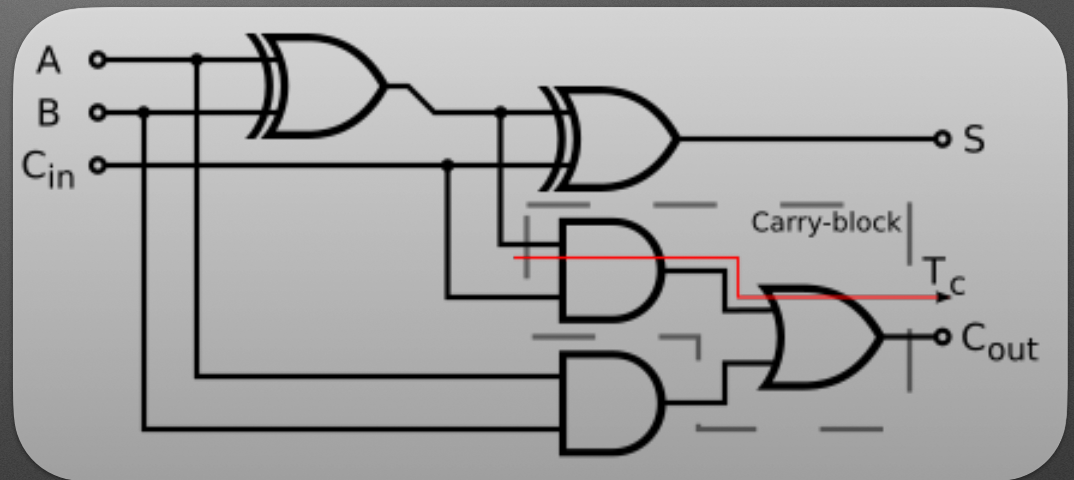
Multi-bit and Ripple Carry

JLS Example: 1111+0001

Ripple Adder

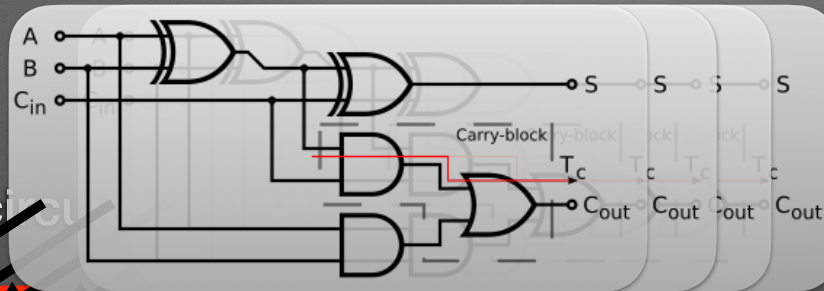
- Example: 1111 + 0001
- As a traditional math problem:

$$\begin{array}{r} 1\ 1\ 1\ 1 \\ +\ 0\ 0\ 0\ 1 \\ \hline \end{array}$$



Info in circuit

- As values in a circuit



0	0	0	0
1	1	1	1
-	0	0	1
-	-	-	-
0	0	0	0

<= Carry $i-1$ connected to carry in of i

<= "A"

<= "B"

<= "Sum"

Info in circuit: Initial

- As values in a circuit:

0	0	0	0	<= Carry $i-1$ connected to carry in of i
1	1	1	1	<= "A"
+	0	0	0	1 <= "B"
0	0	0	0	<= "Sum"

Info in circuit: After 1st “Sum” update

- As values in a circuit:

0	0	1	0	<= Carry $i-1$ connected to carry in of i
1	1	1	1	<= “A”
+	0	0	0	1 <= “B”
1	1	1	0	<= “Sum”

Info in circuit: After 2nd “Sum” update

- As values in a circuit:

0	1	1	0	<= Carry $i-1$ connected to carry in of i
1	1	1	1	<= “A”
+	0	0	0	1 <= “B”
1	1	0	0	<= “Sum”

Info in circuit: After 3rd “Sum” update

- As values in a circuit:

1	1	1	0	<= Carry $i-1$ connected to carry in of i
1	1	1	1	<= “A”
+	0	0	0	1 <= “B”
1	0	0	0	<= “Sum”

Info in circuit: After 3rd “Sum” update

- As values in a circuit:

1	1	1	0	<= Carry $i-1$ connected to carry in of i
1	1	1	1	<= “A”
+	0	0	0	1 <= “B”
0	0	0	0	<= “Sum”

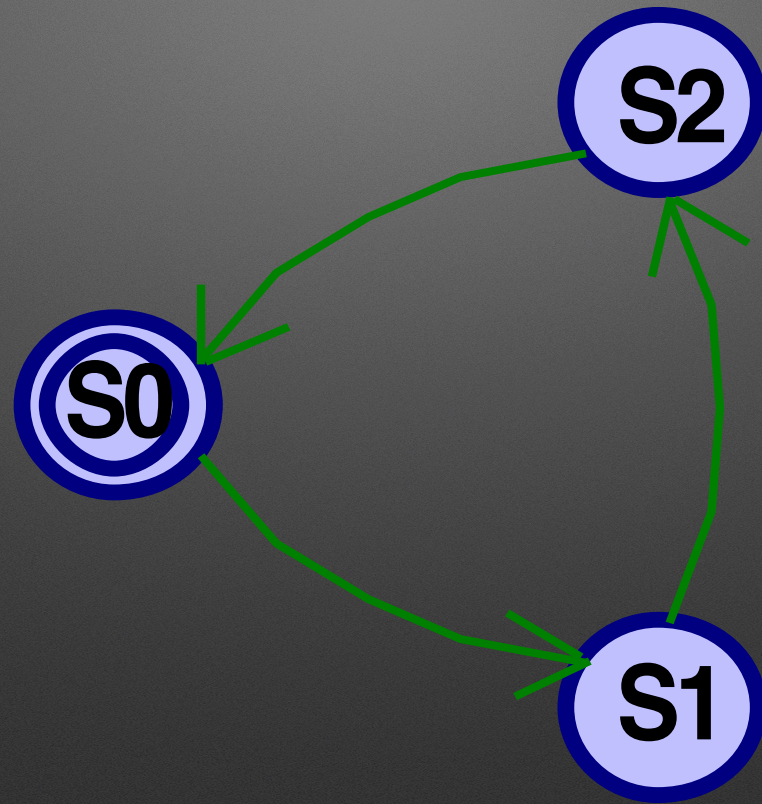
Ripple Adder: Total Time

- N bits: Worst case scenario is ripple through all N
 - If T_c is the propagation delay through the Carry
 $= N \cdot T_c$
 - Dictates things like maximum clock cycle for any paths/loops that use addition
 - Lots of things rely on addition!

Studio 4B: Structural Ripple-Carry Adder (And Generate Statement)

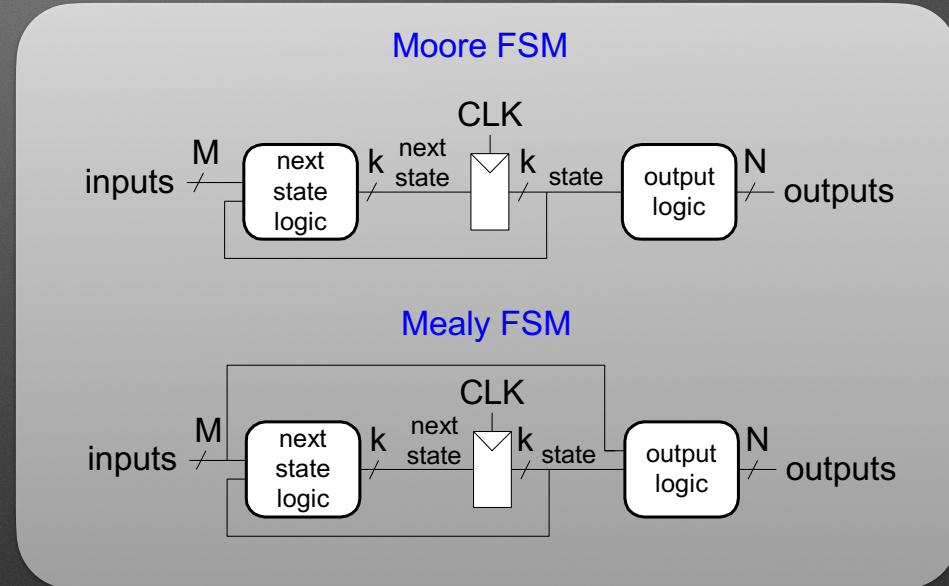
Studio 4B: State Machines

Divide by 3 Counter



Verilog FSMs

- Three parts
 - Next state logic (arrows / next state table)
 - State register (active bubble)
 - Output logic (output equations)



Verilog

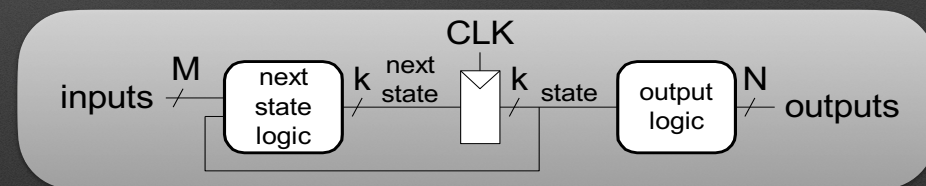
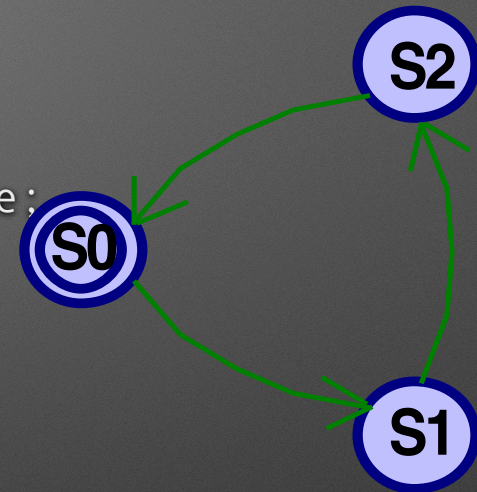
```
module divideby3FSM(input  logic clk,
                    input  logic reset,
                    output logic q);

    typedef enum logic [1:0] {S0, S1, S2} statetype;
    statetype state, nextstate;

    // state register
    always_ff @(posedge clk, posedge reset)
        if (reset) state <= S0;
        else      state <= nextstate;

    // next state logic
    always_comb
        case (state)
            S0: nextstate = S1;
            S1: nextstate = S2;
            S2: nextstate = S0;
            default: nextstate = S0;
        endcase

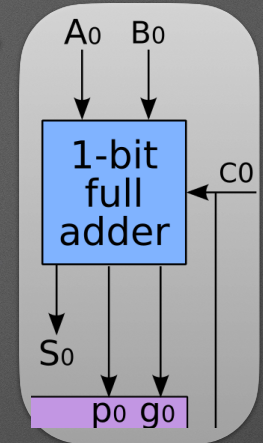
    // output logic
    assign q = (state == S0);
endmodule
```



Aside: Better (faster) Addition

Carry Look-Ahead

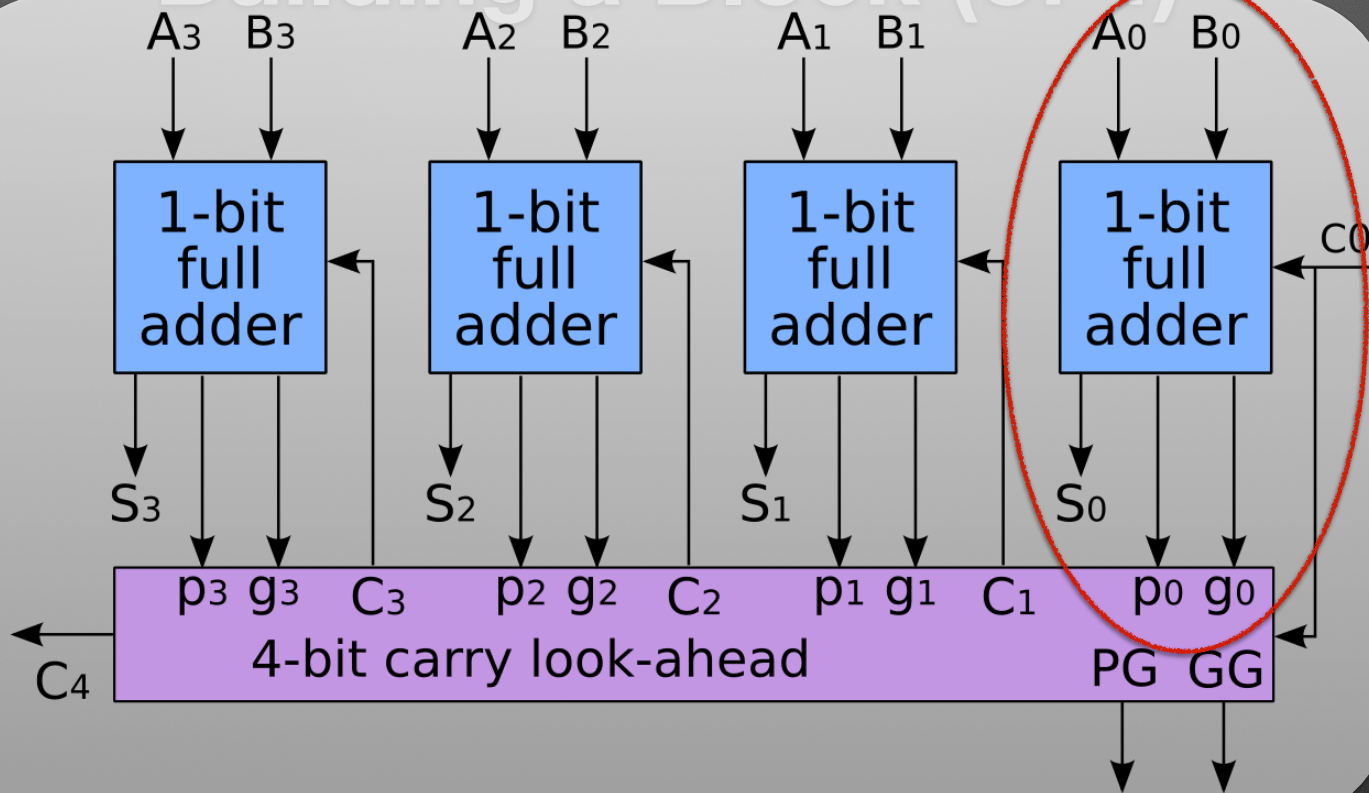
Consider these for the simple full adder



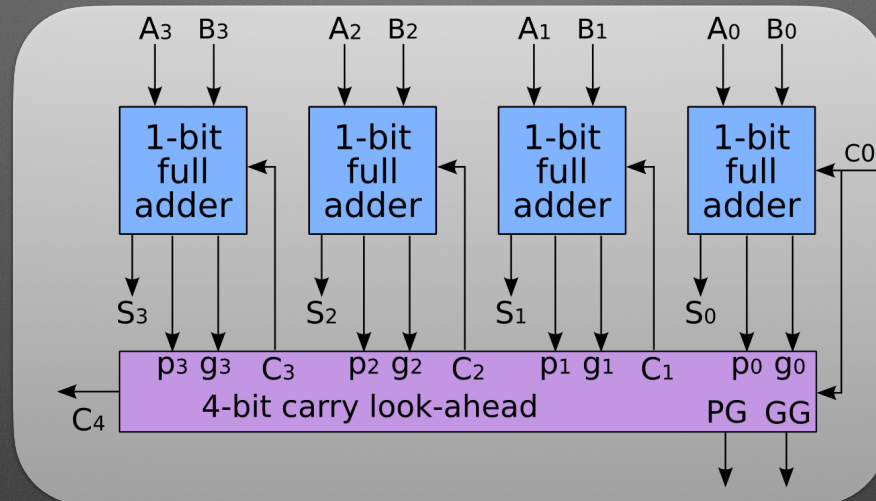
- Divide large addition into n -bit blocks
- *Within each block*, determine what each column would do with a carry-in to the column

?	<= Carry in
a	<= "A"
+ b	<= "B"
— —	
s	<= "Sum"
- Would it "Generate" a carry *no matter what*? (g_x)
- Would it merely "Propagate" the incoming carry? (p_x)
- Can the carry-out be represented in terms of a_x , b_x , cin_x , g_x and p_x ?

Extending Concept: Building a Block (of 4)



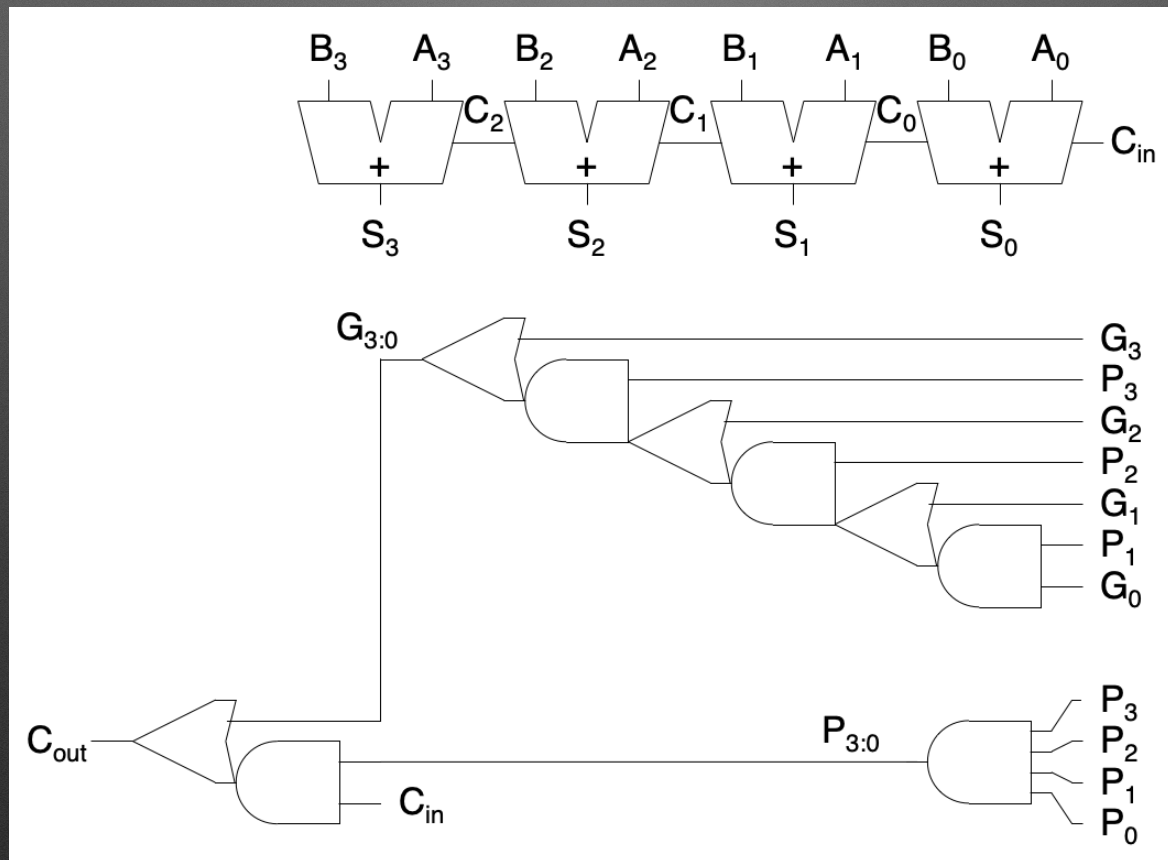
Extend “prediction” to block



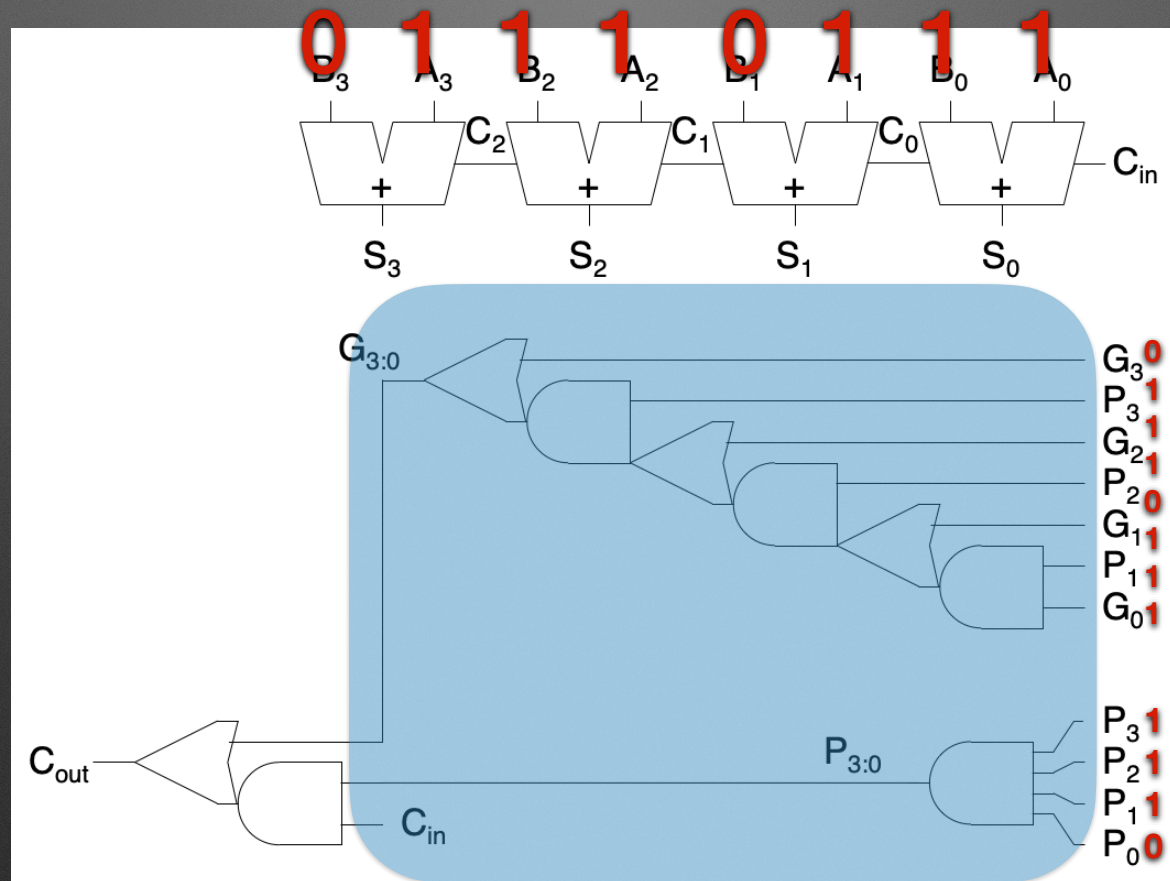
$$P_{block} = P_3 \cdot P_2 \cdot P_1 \cdot P_0$$

$$G_{block} = G_3 + P_3 \cdot (G_2 + (P_2 \cdot (P_1 \cdot G_0)))$$

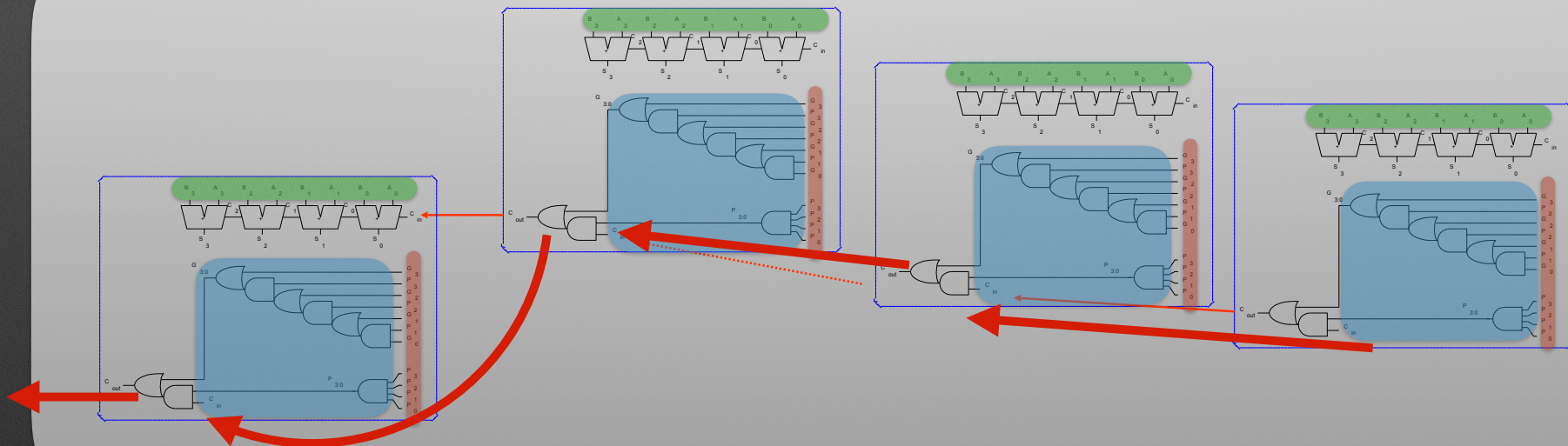
Block “Prediction”



Bits of A & B arrive



Ripple in 4 block, 16-bit CLA



Ripple vs. CLA

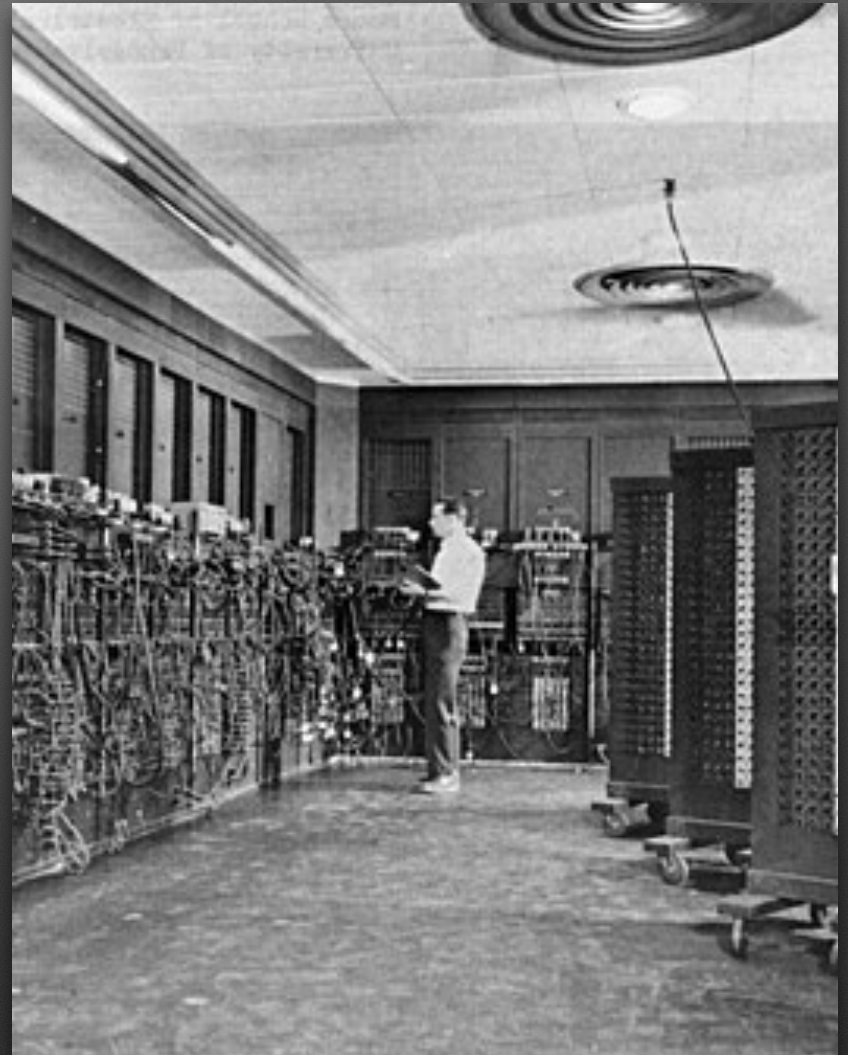
- Ripple
 - Propagation delay is $N_b \cdot T_c$
 - N_b is the number of bits; T_c is the propagation delay of the carry
- **CLA**
 - Propagation delay is $N_{BL} \cdot T_{ao}$
 - N_{BL} is the number of blocks; T_{ao} is the propagation delay of an and+or
 - $N_{BL} < N$. (usually $\frac{1}{2^k}$ the size, where $k \geq 2$)

Trade off: Logic vs. Time

- CLA and other tricks (Prefix adder) add logic to reduce time
- Degenerate case: A look-up table (full sum-of-products equation)
 - How many layers of logic? (nots, ands, ors)?
- To estimate complexity, how many rows and output columns are in a table to add two, 8-bit numbers?
 - Approximately how many AND gates?
Approximately how many OR gates?
Estimate the number of inputs that may be needed on OR gates

Chapter 5

Goal...and pattern...



$$X = Y + Z$$

(Using variables)

$$X = Y - Z$$

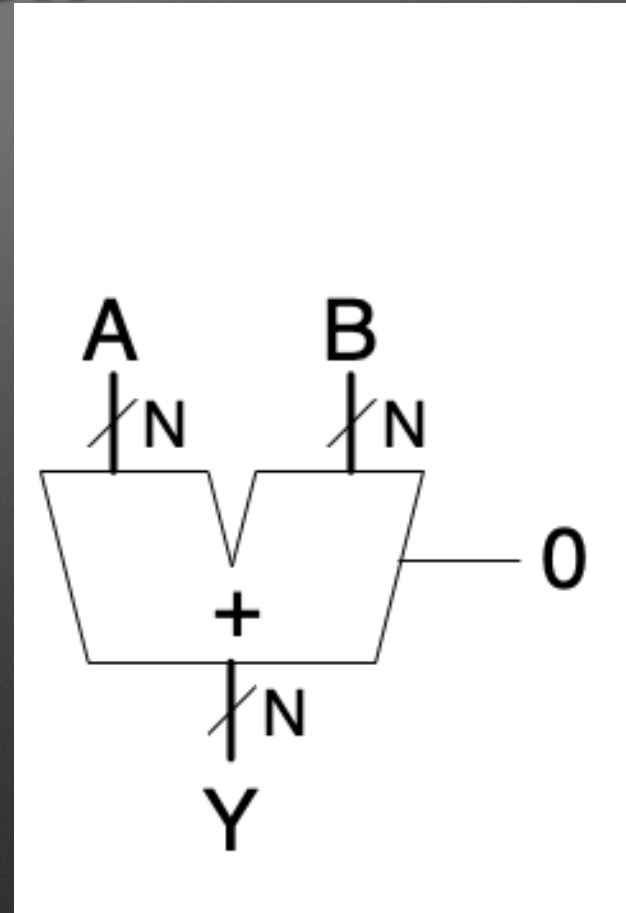
(Using variables)

$X = Y < Z$
(Using variables)

$X = Y \& Z$
(Using variables)

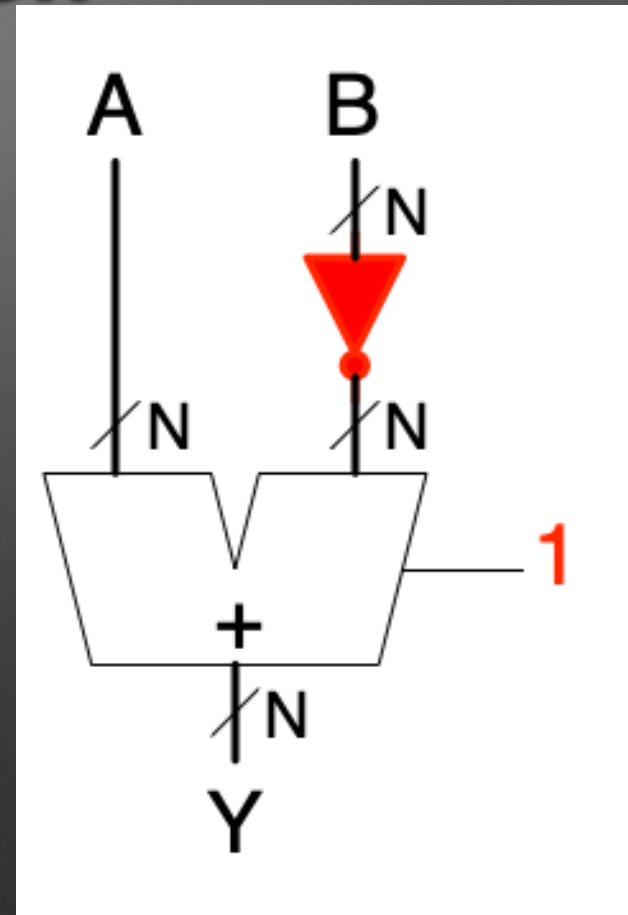
Subtraction

- The beauty of 2's complement
- $A - B = A + \bar{B} + 1$



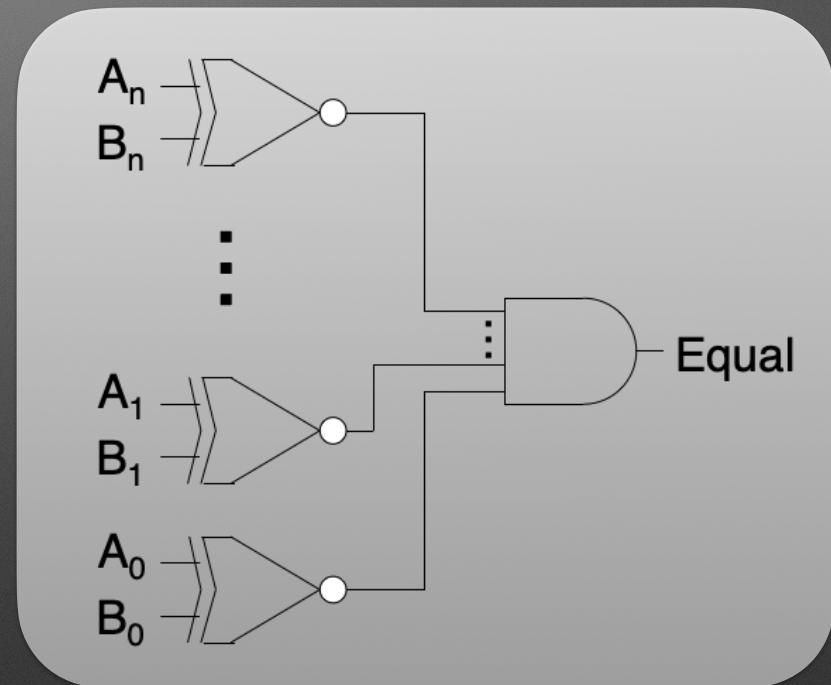
Subtraction

- The beauty of 2's complement
- $A - B = A + \overline{B} + 1$



Comparisons

- Equality
 - Easy: Are any bits different?
- Equal to zero?
- ?

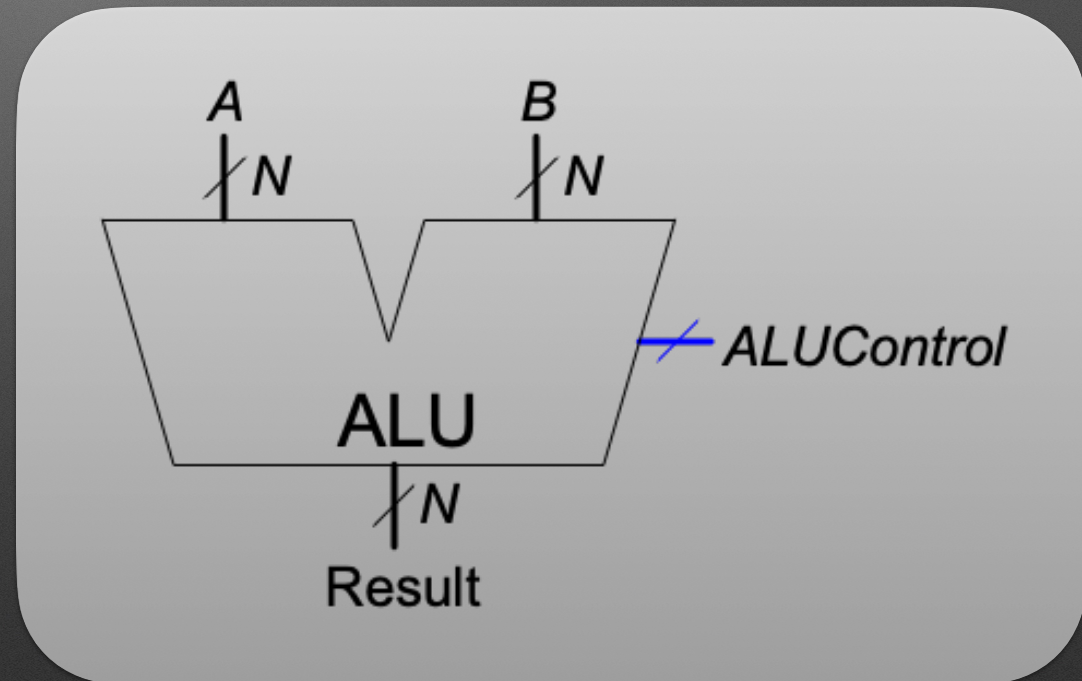


Comparisons

- Less than (signed): Is $A < B$?
- Leverage Subtraction: $A < B$ is equivalent to $A - B < 0$
 - Subtract and check result
 - General: Is $A - B$ negative?
 - But...large numbers can “overflow”.
Need to consider overflow and signs of A & B

ALU: Arithmetic Logic Unit

- “Heart” of CPU: Does the computation stuff.
 - Basic operations
 - Addition
 - Subtraction
 - Bitwise AND
 - Bitwise OR
 - Comparison ($<$)

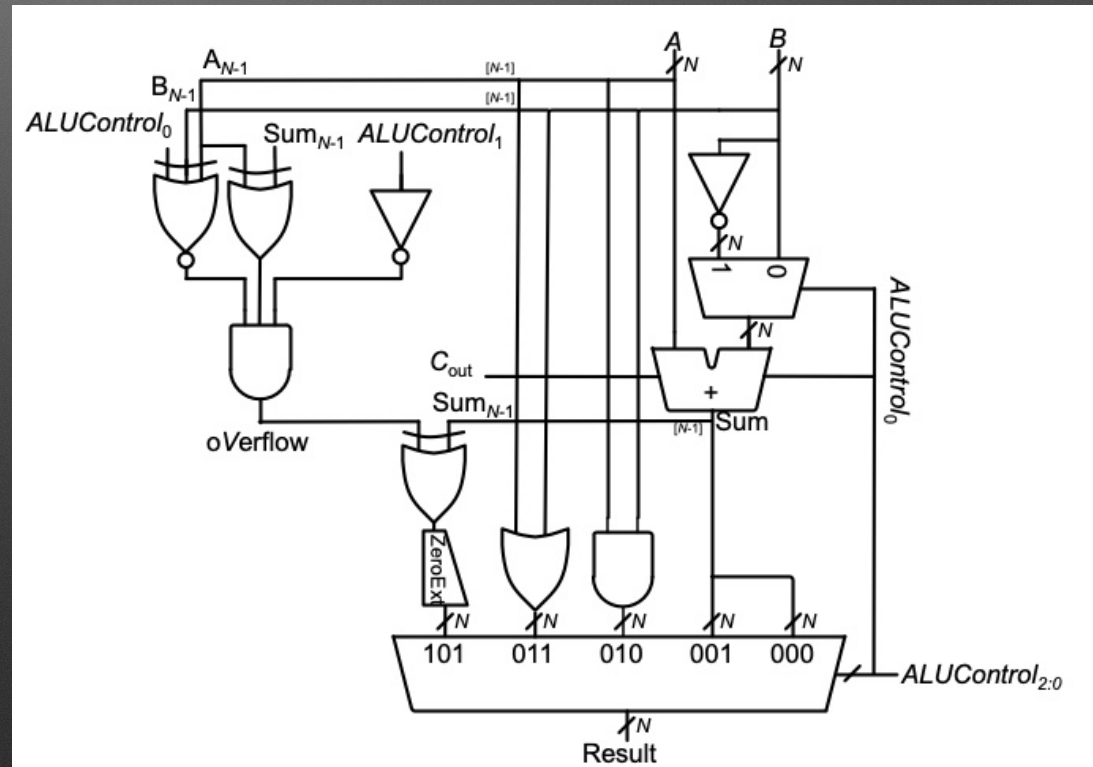


ALU: Arithmetic Logic Unit

- **“Heart” of CPU: Does the computation stuff.**

- **Basic operations**

- Addition: 000
- Subtraction: 001
- Bitwise AND: 010
- Bitwise OR: 011
- Comparison ($\leq$): 101



Other Operations: Shift Left (one place)

- $2^5 + 2^4 + 2^2 + 2^1 = 54$

Value	0	0	1	1	0	1	1	0
Place Value	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- $2^6 + 2^5 + 2^3 + 2^2 = 108$

Value	0	1	1	0	1	1	0	0
Place Value	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Shifting (unsigned / ∞ width)

- Left n bits: Equivalent to multiplication by 2^n
 - Multiplication algorithm is a mix of addition, 1-digit multiplication, and multiplication by b^k
 - Ex: 123×12
 - Ex: $1011_2 \times 11_2$
- Right n bits: Equivalent to division by 2^n and truncation

Other Uses of Shifting (Serial Communication) & State Machines

LED & Key

Memory

Memory / Storage

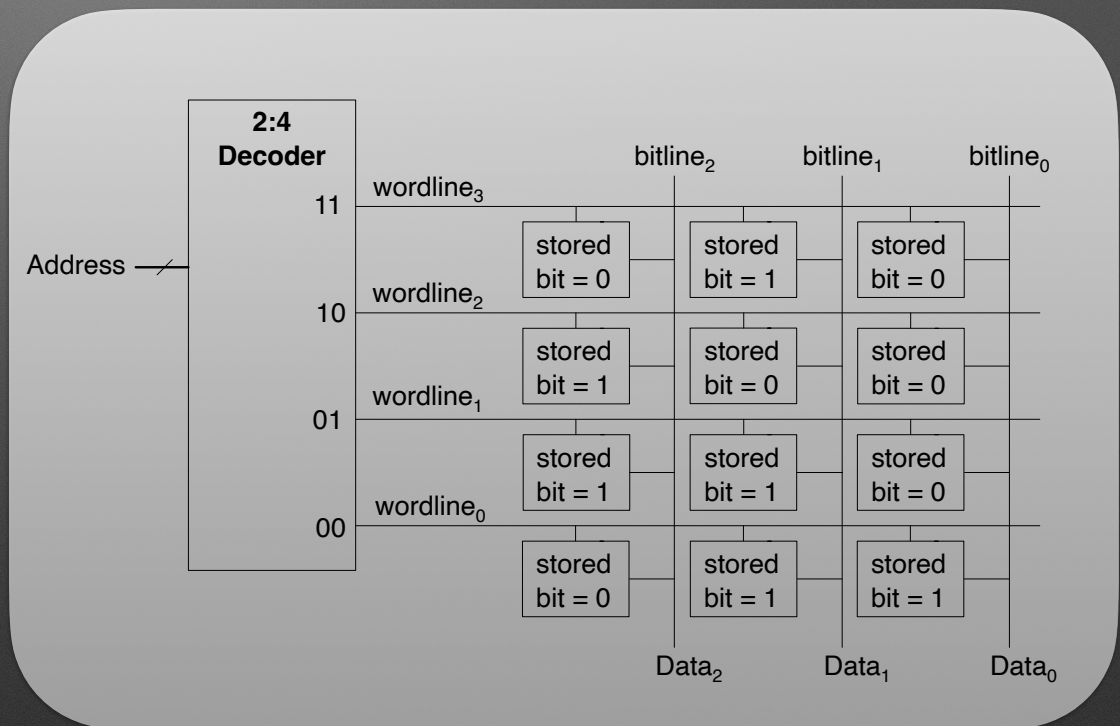
- Common types
 - Static Random Access Memory (SRAM)
 - Dynamic Random Access Memory
 - Read Only Memory (contents can't be easily changed)

Memory / Storage

- General Approach
 - Store in a 2D grid of elements
 - Call each row a “word”
 - Each row has an index to access the content of the entire row

Memory Structure

- One approach
- Bits are “enabled” to connect to shared output line

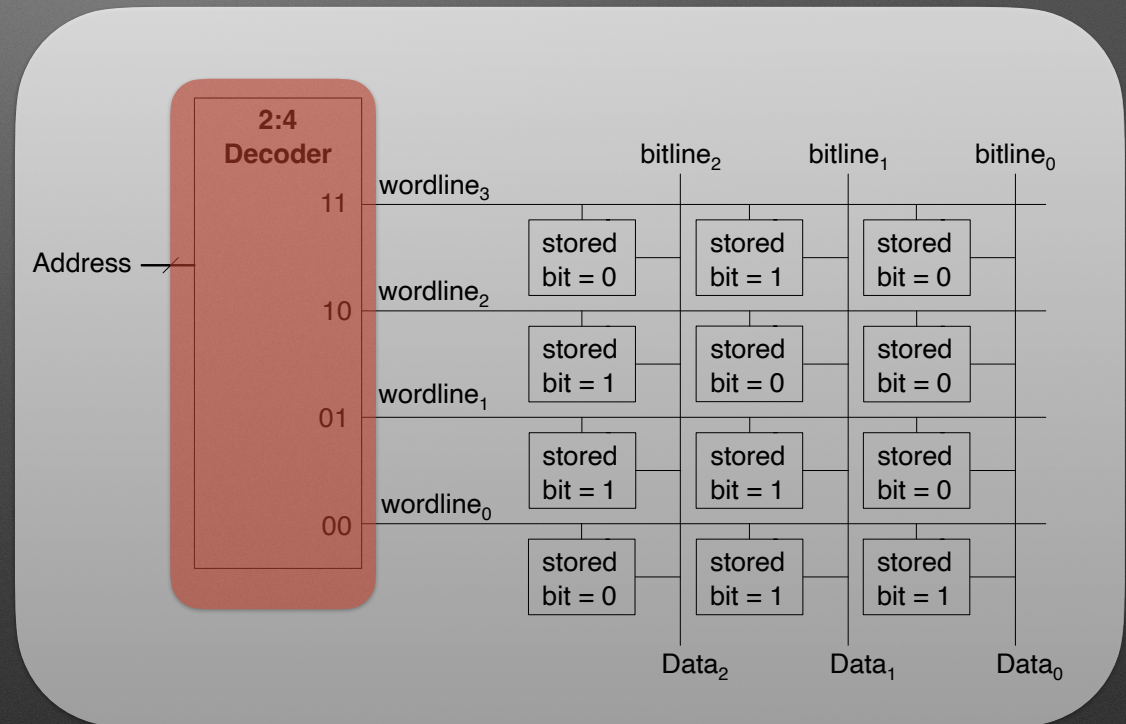


Memory / Storage

- Concept: Computer programming
 - An Array (List) is a representation of memory
- “Random” : Largely about (*unimportance of*) *time to access*
 - The “random” means the location doesn’t have much impact on access time
- Vs. “Sequential”

RAM Memory Structure

- One approach
 - Bits are “enabled” to connect to shared output line



Vs. A Sequential Example



SRAM vs. DRAM

- S = “Static”/Unchanging (well, only changing when requested!)
 - Could be built from D Flip Flops (but similar “self-reinforcing” circuits more likely)
- D = Dynamic: Values fade if not refreshed
- RAM: “Random Access”
 - About performance of reading/updating
 - Time take (*propagation delay*) does not depend on index requested

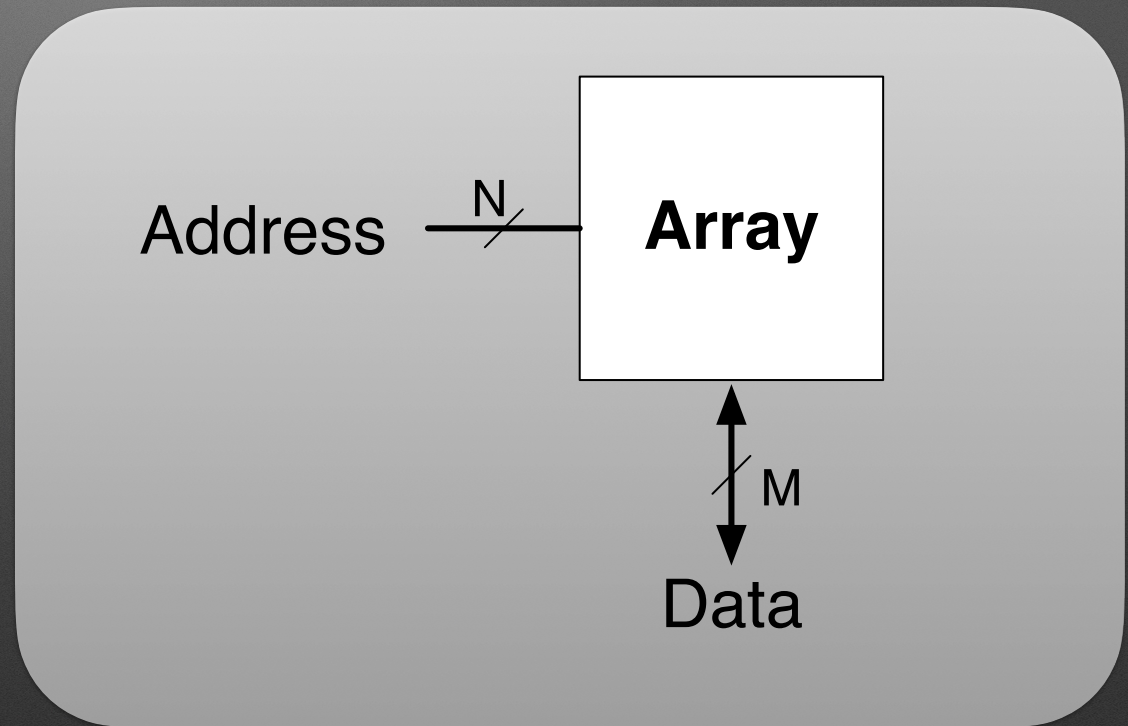
RAM: SRAM vs. DRAM

ROM

- Read Only
 - But still “Random Access” performance
- Fixed look-up table. Could be built with combinational logic!
 - Earlier example of “adder” could just be a ROM
 - LUTs in FPGA are small ROMs

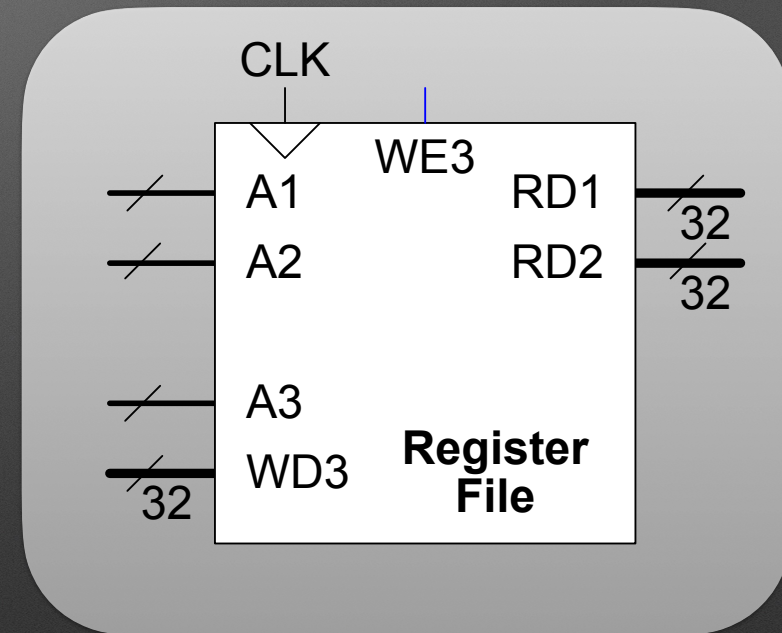
Reading Memory

- M = Word size
- N = “address size”
- How many total bits are stored?



ALU Operations

- Context: $X=Y+Z$
 - We need places to hold Y, Z, and X.
- Need TWO inputs:
 - need a memory structure that provides 2 values (i.e. dual output ports)
- The “Register File”
- Also supports writing (updating)



JLS Register File (W/ D Flip Flops)

Verilog: RISC-V Register File

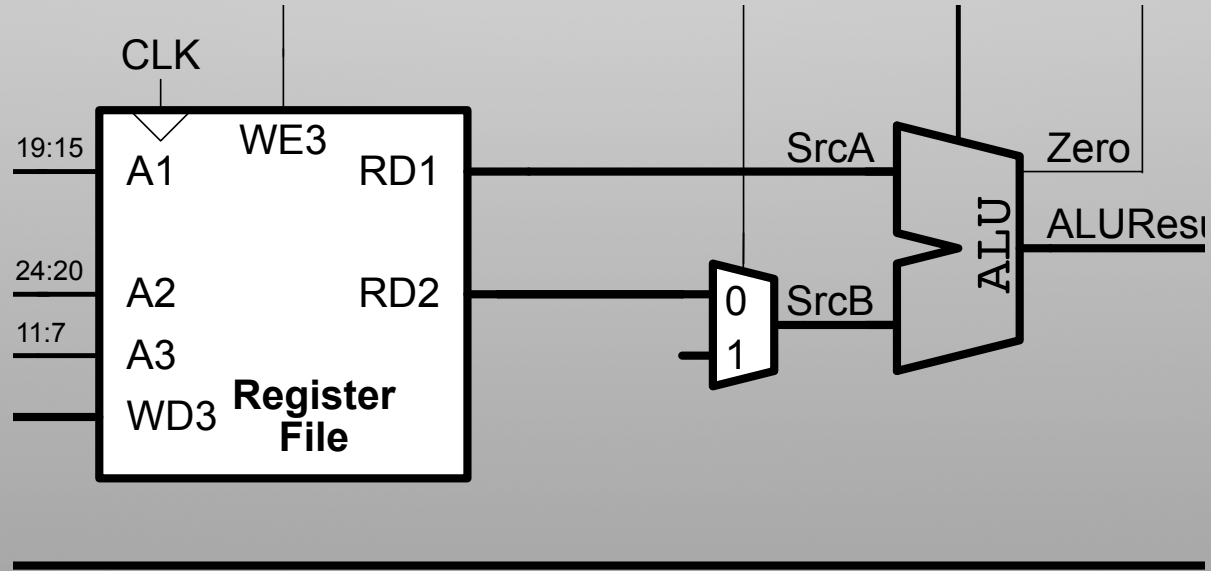
```
// 32 x 32 register file with 2 read, 1 write port
module regfile(input  logic      clk,
               input  logic      we3,
               input  logic [4:0] ra1, ra2, wa3,
               input  logic [31:0] wd3,
               output logic [31:0] rd1, rd2);

    logic [31:0] rf[31:0];

    always_ff @(posedge clk)
        if (we3) rf[wa3] <= wd3;

    assign rd1 = rf[ra1];
    assign rd2 = rf[ra2];
endmodule
```

Big Picture: add x, y, z



FPGA

- Field Programmable
- Gate Array
 - Lattice iCE40 UP5k: Architecture Overview
 - RAMs, (Dual and Single Port)
 - Look Up Tables (LUTs): 4 inputs
 - D Flip Flops
 - Lots: ~5,000

Questions

- Chapter fits well with 3601
 - Wait until next chapter...
- Why so many memory types / what are the differences?
 - Evolution over time
 - Different needs: Capacity vs. Need — the memory hierarchy

Questions

- PLA vs. FPGA
 - PLA: (largely) 2-level logic / simple combinational logic
 - FPGA: Array of many programmable blocks with programmable interconnects
 - Can efficiently achieve more than 2-layer logic
 - Memory/storage is inherent (can do full state machine...see hw 4b)

